Markov Decision Processes

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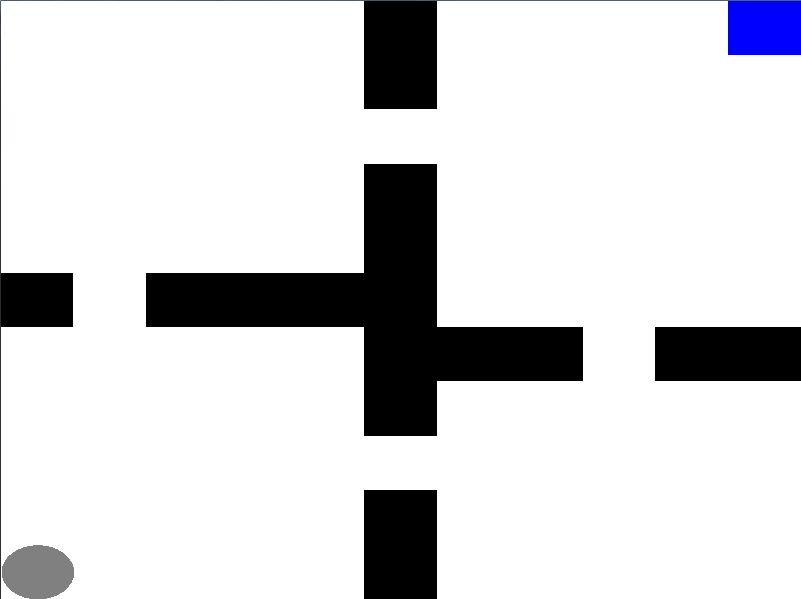
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Introduction

Small Grid World

*Why is it interesting?*

A small grid world with four rooms was used for the small MDP. It is shown in Figure 1. The small grid world has 104 states. The agent starts in the bottom left corner and works its way to the goal state in the top right corner. The transition function is as follows: there is an 80% chance the agent will go in the direction intended and a 6.7% chance of moving in a direction that is not intended. The reward function gives a reward of 100 when the agent reached the goal state. Otherwise, the agent receives a reward of -1 for any other state. This gives the agent motivation to end the game as quickly as possible.



*Figure 1: Small Grid World*

Value Iteration

Decreasing the discount factor decreases the number of iterations to find the optimal policy. But this is because the recursive application of the discount factor is making the difference between the later value functions smaller than delta much earlier than it would with a large discount value. This leads to a weak policy being found which in turn increases the number of iterations it takes for the agent to reach the absorbing state. This is because the optimal policy will only get an optimal value for the first few squares. After that the agent must choose randomly amongst the remaining squares. This can especially be seen with discount=1.

Figure 2 shows a policy map when gamma = 0.99. Here future reward is not being discounted much and thus the reward is able to propagate out from the goal state. This creates a clear policy for the agent to follow.

Figure 3 shows a policy map when gamma = 0.5. Since future reward is discounted by such a large factor the utility is not able to propagate out from the goal state. The red indicates that the utility for those states is very low at about 5. The utility of approaching the goal state is not realized until a few squares before as shown by the blue in the top right corner. This is why some parts of the grid can show the agent wandering in non-ideal directions. The agent does not know about the future reward because it is too far away and has been discounted.

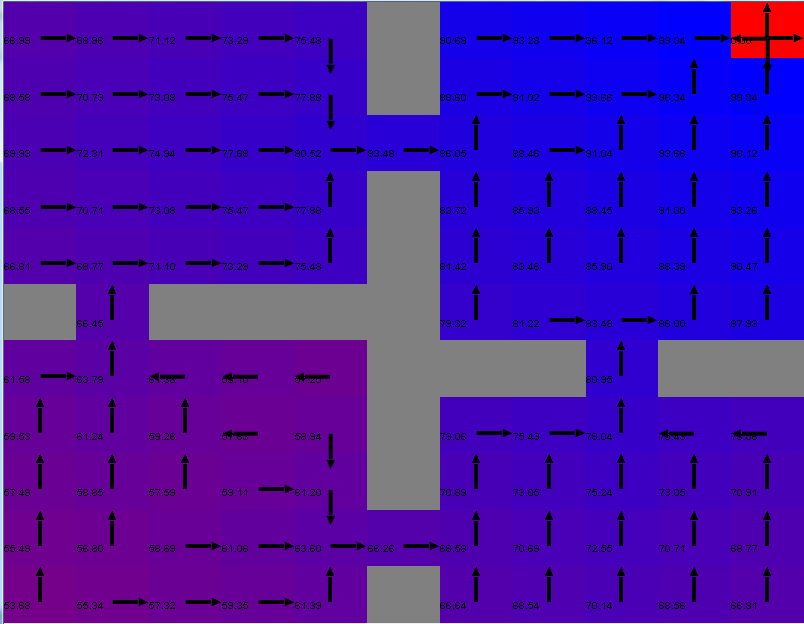
Figure 4 shows a time vs gamma graph for both VI and PI. The figure represents the amount of time needed for the algorithm to converge for a specified gamma. VI outperforms PI all gamma choices greater than 0.6. PI only outperforms VI for 0.5 < gamma < 0.6. The time needed for convergence for VI decreases as gamma increases. At gamma = 0.99 the time needed is approximately 75 ms.

Figure 5 shows an iterations vs gamma graph for both PI and VI. The figure represents the number of iterations needed for convergence for a specified gamma. The VI always has a larger number of iterations no matter the choice of gamma. Also the number of iterations needed is growing linearly with gamma.

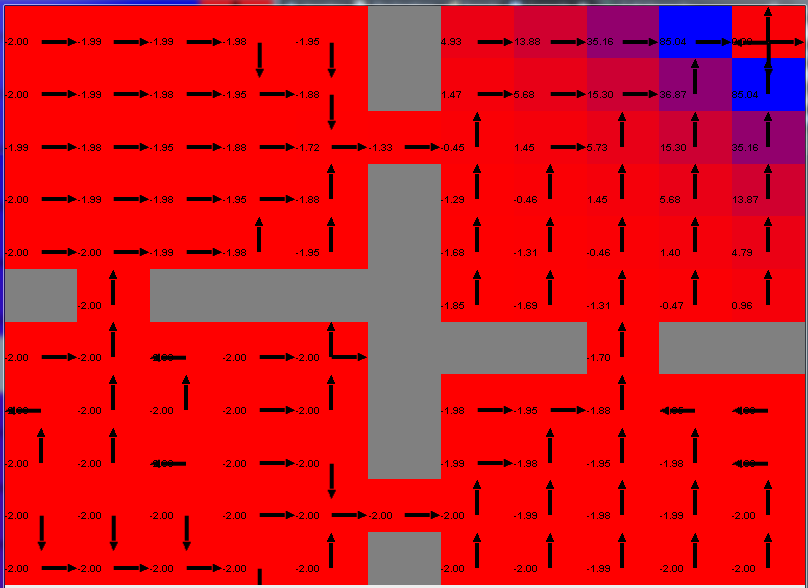
Graph ideas. Passes vs discount factor. Iterations needed to get to goal vs passes or discount factor. Convergence measure vs time

For VI a greedy Q policy is used which plans from the input state and returns a policy. This policy greedily selects actions with the highest Q value and breaks ties uniformly randomly. (See GreedyQPolicy in ValueIteration for more in burlap).

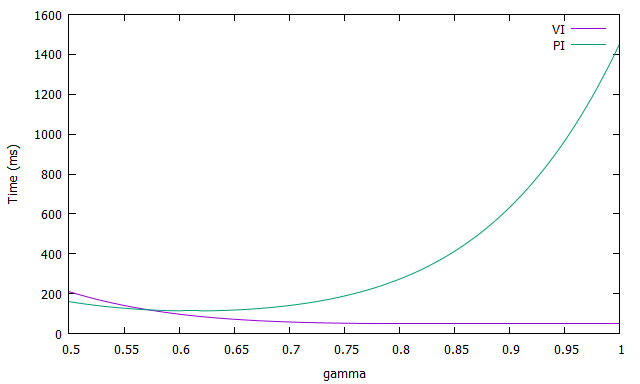
VI and PI converge to the same policy. This means either the VI found the true utilities or utilities that created the correct sequence of actions that led to the policy.



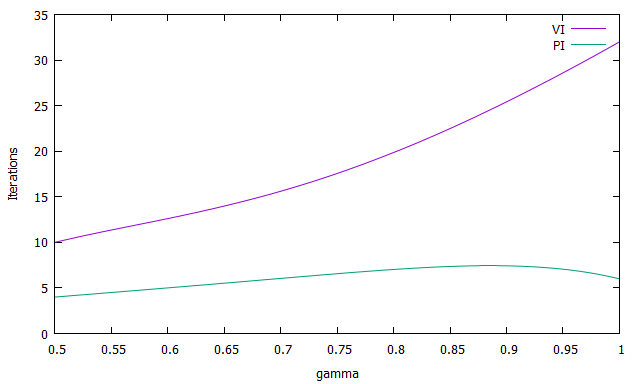
*Figure 2: VI, Policy map for gamma = 0.99*



*Figure 3: VI, Policy map for gamma = 0.5*



*Figure 4: Time vs gamma for PI and VI, Small grid*



*Figure 5: Iterations vs gamma for PI and VI, Small grid*

**Policy Iteration**

Policy iteration typically takes less iterations to converge than value iteration. However, each iteration tends to be more expensive. Each iteration has an inner value iteration that computes a value based on actions defined by the current policy.

For both PI and VI number of iterations increases as gamma decreases. Good for a graph.

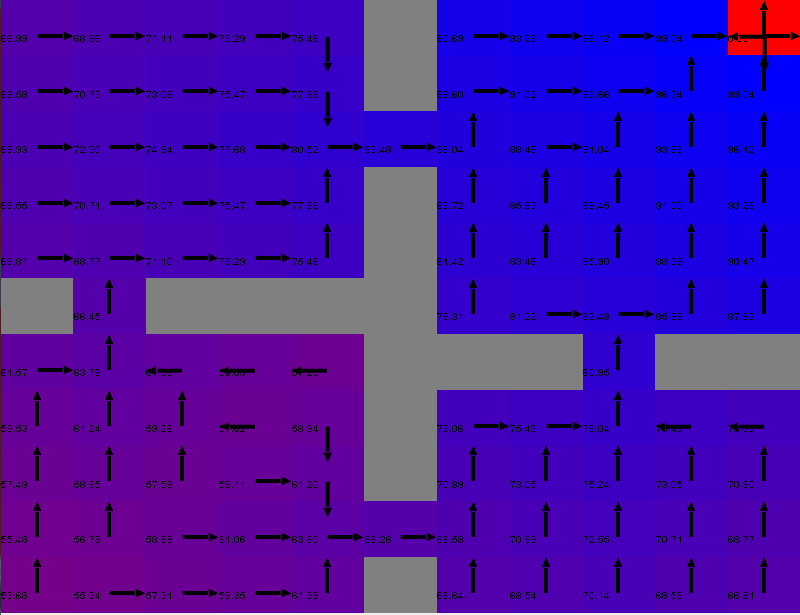
Figure 6 shows the policy map when gamma = 0.99. The policy is almost identical to the policy found by value iteration. The only difference is in the bottom left corner in the starting room. Explain why.

Figure 7 shows the policy map when gamma = 0.5. Similar to VI the small discount factor keeps the utilities from propagating out from the goal state. The policy found is different from the policy found with a larger gamma.

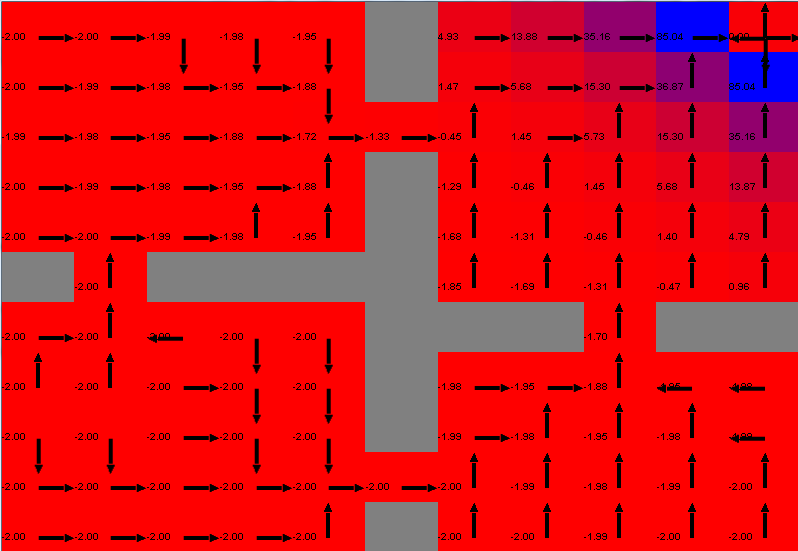
Figure 4 shows the time analysis for PI. PI is only able to outperform VI when 0.5 < gamma < 0.6. The time needed appears to grow exponentially as gamma approaches 1. Why does this happen? How is discount factor affecting this? The time needed when gamma = 0.99 is approximately 1500 ms compared to VI’s 75 ms. This is an increase by a factor of 20. Explain why I’m using gamma = 0.99 for all my choices.

Figure 5 shows the iteration analysis for PI and VI. PI takes many fewer iterations than VI. However these iterations tend to be more time expensive because of the large amount of work being done. Each iteration has several, possibly hundreds, of value iteration calculations. Each calculation runs a value iterations based on actions of the current policy. Thus each inner value iteration is not as computationally expensive because the computations are only needed for one action per state (this may not be right). The system of equations being solved is linear. Normal VI has a non-linear system of equations. The number of iteration needed by PI when gamma = 0.99 is 6 compared to the 33 iterations needed by VI. The PI curve stays relatively level no matter the gamma choice with either 5 or 6 iterations being needed.

Show the average time per iteration for VI and PI. PI iterations take longer and thus have a higher average. However, each iteration takes less time than the previous because it requires less value evaluations.



*Figure 6: PI, Policy map for gamma = 0.99*



*Figure 7: PI, Policy map for gamma = 0.5*

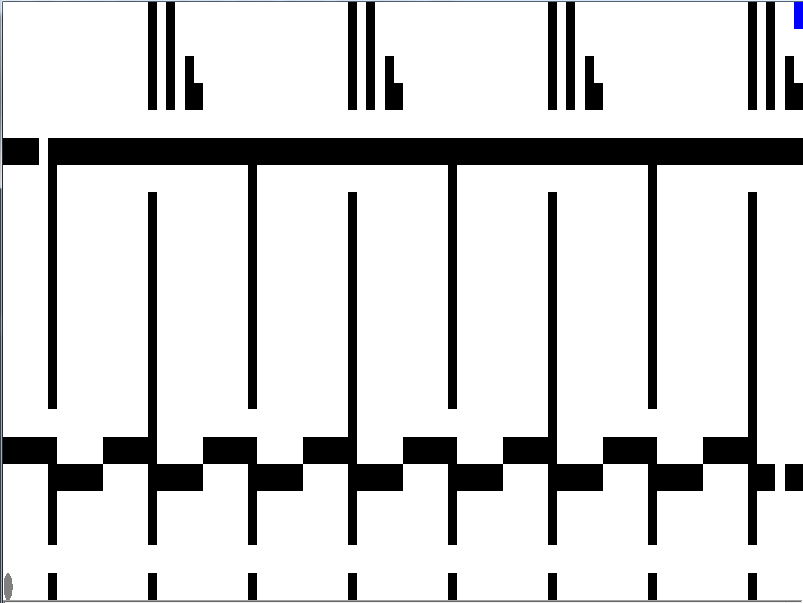
**Large Grid World**

***Why is it Interesting?***

The large grid world is shown in Figure… The agent starts in the bottom left corner and is trying to reach the goal state in the top right corner marked by the blue square.

In the middle portion the agent must make its way through a winding section. The agent takes significantly more time through the winding section than when the section was just a straight travel through.

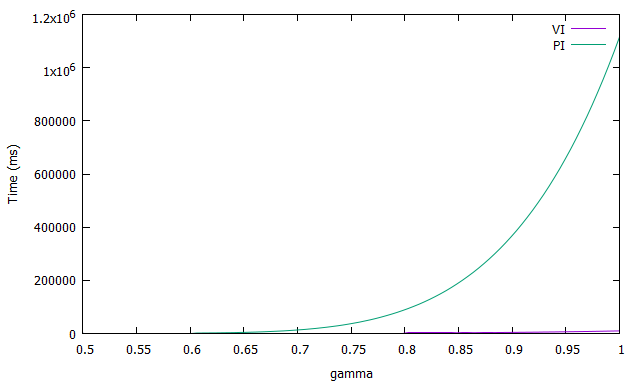
Talk about rewards fir reaching goal state etc.



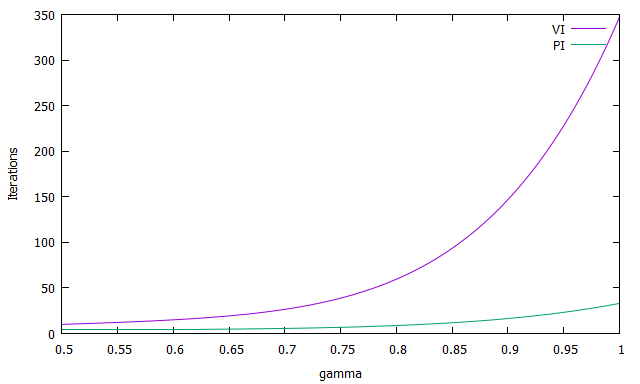
*Figure : Large grid world*

**Value Iteration**

At gamma = 0.99 VI takes approximately 9.7 seconds and PI takes 1114.7 seconds.



*Figure : Time vs gamma, Large grid world*



*Figure : Iterations vs. gamma, Large grid world*

**Policy Iteration**

Policy iteration can take a long time even though the equations for calculating the utilities are linear. When linear equations are being computed their matrixes are inverted. This process can still take a long time for large matrices. We have large matrices when we have a large number of states.

PI is usually faster because we do not need to find true utilities to find the policy. Why is that not working in this case?

PI is guaranteed to converge because the value iterations are always improving our estimate of the policy and there are a finite number of policies. If PI does not converge in this case it is because it is too computationally expensive.

Need to get O notation time needed.